SOLUȚIE

Enunţ: *Fie numerele* $x, y, z > 0$ *astfel încât* $x^2 + y^2 + z^2 = 1$. *Atunci să se demonstreze că*

$$\left[\frac{x-1}{3}\right]+\left[\frac{y+1}{3}\right]+\left[\frac{z}{3}\right]+\left[\frac{2x+1}{6}\right]+\left[\frac{2y+5}{6}\right]+\left[\frac{2z+3}{6}\right]\le\frac{2\sqrt{3}}{3}.$$

Soluţie: Se observă că: $\frac{x-1}{3}+\frac{1}{2}=\frac{2x+1}{6}$, $\frac{y+1}{3}+\frac{1}{2}=\frac{2y+5}{6}$ şi $\frac{z}{3}+\frac{1}{2}=\frac{2z+3}{6}$, astfel că

$$\left[\frac{x-1}{3}\right]+\left[\frac{y+1}{3}\right]+\left[\frac{z}{3}\right]+\left[\frac{2x+1}{6}\right]+\left[\frac{2y+5}{6}\right]+\left[\frac{2z+3}{6}\right]$$

$$=\left[\frac{x-1}{3}\right]+\left[\frac{y+1}{3}\right]+\left[\frac{z}{3}\right]+\left[\frac{x-1}{3}+\frac{1}{2}\right]+\left[\frac{y+1}{3}+\frac{1}{2}\right]+\left[\frac{z}{3}+\frac{1}{2}\right].$$

Folosind identitatea Hermite: $[a]+\left[a+\frac{1}{2}\right]=[2a]$, $a\in\mathbb{R}$ se obţine

$$\left[\frac{x-1}{3}\right]+\left[\frac{y+1}{3}\right]+\left[\frac{z}{3}\right]+\left[\frac{2x+1}{6}\right]+\left[\frac{2y+5}{6}\right]+\left[\frac{2z+3}{6}\right]=\left[\frac{2x-2}{3}\right]+\left[\frac{2y+2}{3}\right]+\left[\frac{2z}{3}\right]\le$$

$$\le\frac{2x-2}{3}+\frac{2y+2}{3}+\frac{2z}{3}=\frac{2(x+y+z)}{3}\le\sqrt{\frac{x^2+y^2+z^2}{3}}=\frac{2\sqrt{3}}{3}.$$